

The study of the nucleus-nucleus interaction potential for $^{16}\text{O}+^{27}\text{Al}$ and $^{16}\text{O}+^{28}\text{Si}$ fusion reactions

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Abstract

Using the Monte Carlo simulation method accompanied by the modifying effects of the density distributions overlapping, we have examined the nuclear matter incompressibility effects for asymmetric systems with light nuclei, namely $^{16}\text{O}+^{27}\text{Al}$ and $^{16}\text{O}+^{28}\text{Si}$ fusion reactions. The obtained results show that the nuclear equation of state has considerable influence on the calculation of fusion probabilities for these asymmetric systems.

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1. INTRODUCTION

The recent fusion experiments using Medium-Heavy nuclei show remarkable changes in the fusion excitation functions (FeF) at deep sub-barrier energies [1, 2]. Further research indicated that these changes can be attributed to the hindrance effects in fusion reactions [3]. In theoretical models which are based on the sudden approximation to calculate the nuclear potential, one can explain the observed changes in slope of the FeF by using the simulation of repulsive core effects. In fact these modifications take into account the effects of Pauli Exclusion Principle in calculation of nuclear potential and correct it in the inner regions [4, 5]. In general, the shallow potential resulting from the modified M3Y interaction with repulsive core can explain fusion cross sections at deep sub-barrier energies for symmetric (asymmetric) systems with positive Q-value [4]. This research has been done on symmetric Medium-Medium nuclei fusion reactions such as $^{58}\text{Ni}+^{58}\text{Ni}$ and $^{64}\text{Ni}+^{64}\text{Ni}$ [4, 6], Light-Heavy nuclei such as $^{16}\text{O}+^{208}\text{Pb}$ [5] and symmetric Light-Light nuclei such as $^{16}\text{O}+^{16}\text{O}$ [7] successfully. So we have motivated to investigate the hindrance effects on asymmetric Light-Light reactions at energies above the fusion barrier. For this purpose we have selected $^{16}\text{O}+^{27}\text{Al}$ and $^{16}\text{O}+^{28}\text{Si}$ reactions which their Q-values for producing the compound nucleus are positive.

In Sec. 2 we shall briefly discuss about the employed model in the calculation of total interaction potential and correction of this model by using an additional repulsive force in the form of a repulsive core for modeling the hindrance effects. Calculation process of potential and fusion cross section for selected reactions is given in Sec. 3. Section 4 is devoted to some concluding remarks.

2. THEORY

A. Internuclear Potential

The interaction potential between two nuclei is given by,

$$V(r) = V_C(r) + V_N(r), \quad (1)$$

where $V_C(r)$ and $V_N(r)$ are resulting from the large-range Coulomb repulsion and short-range nuclear attraction in nucleon-nucleon (NN) interactions, respectively. The nuclear potential between target and projectile nuclei plays a crucial role in the study of fusion cross section. In order to calculate the nuclear part of total potential we have employed a simulation method that it has reported in our previous work [8]. In this simulation technique, we have used the neutrons and protons densities for calculation of nuclear potential. This capability allows us to consider the effects of the surface nucleons in the calculation of interaction potential [9].

B. Modification of Internuclear Potential

Double folding (DF) model, because of using sudden approximation, predicts negative values for inner regions of the total potential (see Fig. 1). When two interacting nuclei begin to overlap and nuclear matter density roughly becomes twice the saturation density ($\rho \approx 2\rho_0$), it could be predicted that an extra repulsive force prevents the compressing of nucleus. In order to consider the modifications of the incompressibility effect on the calculation of the nuclear potential, it is suggested that this can be simulated by adding a zero-range interaction as the following form

$$v_{rep}(\mathbf{s}) = V_{rep}\delta(\mathbf{s}), \quad (2)$$

to the nucleon-nucleon interaction [6], where V_{rep} is a constant parameter. In fact, this interaction simulates the Pauli Exclusion Principle effect on the calculation of nuclear potential. The constants of this repulsive force can be calculated by using the nuclear part of the total potential when two interacting nuclei overlap completely. It has been shown that the value of V_N at $r = 0$ can be estimated using the following relation [6],

$$V_N(r = 0) = \Delta V \approx \frac{A_P}{9}K, \quad (3)$$

where A_P is the mass number of smallest nucleus in the case of an asymmetric system and K is the incompressibility constant and is given by the following relation,

$$K = 9 \left(\rho^2 \frac{\partial^2 \varepsilon}{\partial \rho^2} \right)_{\rho=\rho_0}. \quad (4)$$

where $\varepsilon(\rho)$ is the binding energy per particle of nuclear matter. We have used Thomas-Fermi model to calculate $\varepsilon(\rho)$ [10]. The nuclear part of total potential can also be calculated using the following relation,

$$V_N(r = 0) = V_{dir}(r = 0) + V_{exc}(r = 0) + \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_1(\mathbf{r}_1) V_{rep} \delta(\mathbf{r}_{12}) \rho_2(\mathbf{r}_2), \quad (5)$$

where V_{exc} and V_{dir} are direct and exchange parts of nuclear potential due to the M3Y force. In computation of the integral part of Eq. (5), the parameter V_{rep} and the diffuseness constant of density distributions of target and projectile nuclei, assuming $a_T = a_P = a_{rep}$, adjusted such that the calculated value of nuclear potential at $r = 0$ and the height of the fusion barrier are in the agreement with the predicted values from the Eq. (3) and the corresponding experimental data for each reaction, respectively.

C. One-Dimensional Barrier Penetration Model (ODBPM)

To study of fusion cross section, we have used the ODBPM [11, 12]. In This formalism, the cross section for complete fusion is given by

$$\sigma_{fus}(E) = \sum_{\ell=0}^{\infty} \sigma_{\ell}(E), \quad (6)$$

where the partial-wave cross sections can be calculated using

$$\sigma_{\ell}(E) = \frac{\pi \hbar^2}{2\mu E} (2\ell + 1) T_{\ell}(E). \quad (7)$$

In this relation, μ is the reduced mass of the projectile and target systems and $T_{\ell}(E)$ is the transmission coefficient for angular momentum ℓ through potential barrier at center-of-mass energy E . This latter coefficient can be computed using the WKB [13] approximation for penetration through the barrier,

$$T_{\ell}(E) = \left[1 + \exp \left(2 \sqrt{\frac{2\mu}{\hbar^2}} \int_{r_{1\ell}}^{r_{2\ell}} dr \left[V_0(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} - E \right]^{1/2} \right) \right]^{-1}, \quad (8)$$

where $r_{1\ell}$ and $r_{2\ell}$ are classical turning points for angular momentum ℓ and $V_0(r)$ is total potential for $\ell = 0$. We refer the reader to Ref. [11-13], where this formalism is completely explained.

3. CALCULATIONS

For fusion reactions with light interaction nuclei, because the radius of the target and projectile nuclei are small, one expects that the nuclear matter incompressibility effects have less importance in the calculation of total potential, particularly in place of forming Coulomb barrier. In order to investigate the effect of nuclear matter incompressibility on light fusion reactions, we have selected two systems $^{16}\text{O} + ^{27}\text{Al}$ and $^{16}\text{O} + ^{28}\text{Si}$. Because mass numbers of target and projectile nuclei in each of these reactions are smaller than or equal to 28 and also their Q-values are positive (see Table 2). In the beginning, we have calculated the total potential for the above reactions using the Monte Carlo simulation method and NN interaction of M3Y-Paris type [8]. The density distributions of target and projectile nuclei are parameterized by using the two-parameter Fermi-Dirac distribution functions such that,

$$\rho_i(r) = \frac{\rho_0}{1 + \exp [(r - R_{0i})/a]}. \quad (9)$$

The values of diffuseness $a_{o(n,p)}$ and radius $R_{o(n,p)}$ parameters for proton and neutron densities, that are obtained by Hartree-Fock-Bogoliubov (HFB) calculations [14], have been listed in Table 1. The calculated total potentials for $^{16}\text{O} + ^{27}\text{Al}$ and $^{16}\text{O} + ^{28}\text{Si}$ reactions have been shown in Fig. 1. As a result, the nuclear potential using the M3Y nucleon-nucleon interaction results the negative values for the inner part of the total potential. With adding the repulsive potential to the NN interaction one can take into account the effects of the nuclear matter incompressibility on the nuclear potential calculation. This correction is performed using the method introduced in the previous section and its results are shown in Fig.

1 by M3Y+repulsion potential. Our manner for calculation of parameters used in repulsive core modeling is the fusion cross sections resulting from the M3Y+rep potential have a good agreement with their corresponding experimental data [15, 16]. The obtained values for these coefficients and incompressibility constant K as well as the values of shallow pocket, V_{pocket} , for each reaction have been listed in Table 2. As it is shown in Fig. 1, modifying of M3Y potential leads to appearance of shallow pocket in the inner regions of the total potential and consequently improves the fusion cross section agreement with experimental data (see Fig. 2). Since our aim in this paper is investigation of the compressibility effects on the calculation of fusion cross sections for Light-Light nuclei at energies above the fusion barrier, we have ignored the coupled-channel effects in the calculation of cross sections.

4. RESULT AND DISCUSSION

In this paper we have investigated the dependence of the total interaction potential on the nuclear equation of state for the fusion reactions with asymmetric Light-Light nuclei. For this purpose, we have calculated the nuclear potential for $^{16}\text{O} + ^{27}\text{Al}$ and $^{16}\text{O} + ^{28}\text{Si}$ systems using the Monte Carlo method and M3Y nucleon-nucleon force and modified M3Y+DF potential (M3Y+repulsion). The obtained results are shown in Fig. 1, where the vertical lines show the distance that total overlapping density increases roughly that of normal matter, i.e., $\rho_0 = 0.161 \text{ fm}^{-3}$. As one can see, considering the modifying effects of nuclear matter incompressibility on the calculation of total potential causes increase of the height and thickness of Coulomb barrier as well as appearing shallow pocket in the inner regions of interaction potential. In present work, we have used the ODBPM for computation of the fusion cross section. As a result, the increase of thickness of fusion barrier causes decreasing of penetration coefficient and consequently fusion cross sections. It is evident from Fig. 2 that the obtained results for fusion cross sections resulting from the M3Y+repulsion potential, in comparison with the M3Y potential, are in the better agreement with the corresponding experimental data in above energies. This subject shows that in fusion reactions with light target and projectile nuclei, in spite of the littleness of interaction nuclei radius in comparison with heavy-nuclei, nuclear equation of state can be important in study of light fusion reactions.

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Table 1. The values of the diffuseness $a_{o(p,n)}$ and radius $R_{o(p,n)}$ parameters for proton and neutron density distributions of ^{16}O , ^{27}Al and ^{28}Si nuclei which determined by HFB calculation [14].

Nucleus	R_{0p} (fm)	a_{0p} (fm)	R_{0n} (fm)	a_{0n} (fm)
^{16}O	2.6986	0.4469	2.6519	0.4602
^{27}Al	3.1595	0.4646	3.1361	0.4782
^{28}Si	3.1984	0.4750	3.1671	0.4726

Table 2. The obtained results for V_{rep} , a_{rep} , K and V_{pocket} for $^{16}\text{O} + ^{27}\text{Al}$ and $^{16}\text{O} + ^{28}\text{Si}$ reactions. The Q-values of reactions for the production of the compound nucleus are listed in the last column.

Reaction	a_{rep} (fm)	V_{rep} (MeV.fm ³)	K (MeV)	V_{pocket} (MeV)	Q-value (MeV)
$^{16}\text{O} + ^{27}\text{Al}$	0.325	445.6	234.22	0.48	+14.254
$^{16}\text{O} + ^{28}\text{Si}$	0.340	454.0	234.44	3.94	+11.318

Figure 1:

Fig. 1 The calculated total potentials for (a) $^{16}\text{O}+^{27}\text{Al}$ and (b) $^{16}\text{O}+^{28}\text{Si}$ fusion reactions. The dashed and dotted curves are based on the M3Y and M3Y+repulsion potential, respectively. Vertical line shows the place where the nuclear matter density due to the overlapping of interacting nuclei increases to values about twice the nucleon matter saturation density.

Figure 2:

Fig. 2 The calculated complete fusion cross sections for (a) $^{16}\text{O}+^{27}\text{Al}$ and (b) $^{16}\text{O}+^{28}\text{Si}$ fusion reactions. The experimental data have been extracted from Refs. [15] and [16] for $^{16}\text{O}+^{27}\text{Al}$ and $^{16}\text{O}+^{28}\text{Si}$ systems, respectively .



